

Precise Flattening of Cubic Bézier Segments

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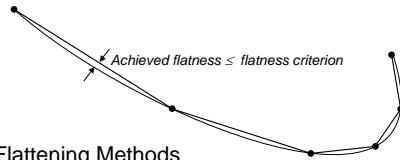


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2D Curve Rendering



- Flattening Methods
 - Forward differencing
 - Recursive subdivision
 - Parabolic Approximation (current method)



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Comparison of Flattening Techniques

- Forward Differencing
 - Uniform interval in t
 - Too many or too few segments
- Recursive subdivision (RS)
 - Conservative achieved flatness
 - Requires achieved flatness evaluation
- Parabolic approximation (PA)
 - Minimal number of segments
 - Faster than RS

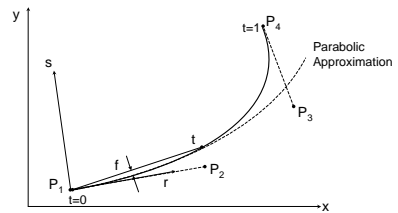


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Approximating the start of a Bézier curve

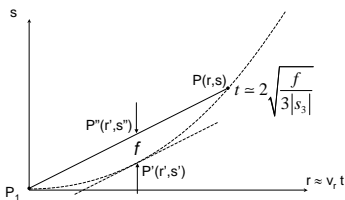


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Parabolic approximation

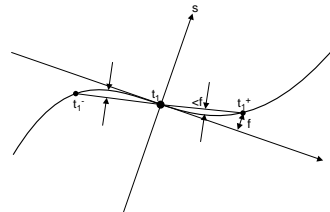


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Inflection Point



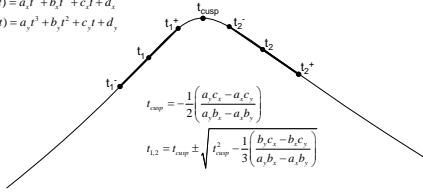
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Inflection Points

$$\begin{cases} x(t) = a_1t^3 + b_1t^2 + c_1t + d_1 \\ y(t) = a_2t^3 + b_2t^2 + c_2t + d_2 \end{cases}$$



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Performance

- Performance measures
 - Relative number of generated segments
 - Relative execution time (C code)
- Test data (curve segment) set
 - Control points at (1,0), (0,0), and (0,1)
 - 4th control point on 100x100 grid (-3 to +3)
 - Flatness criterion at 0.0005

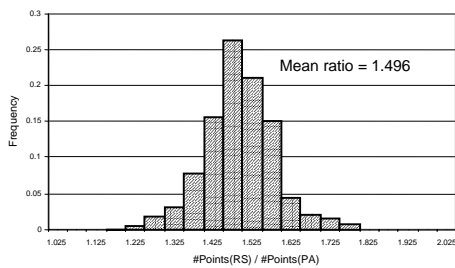


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Ratio of generated segments (RS/PA)

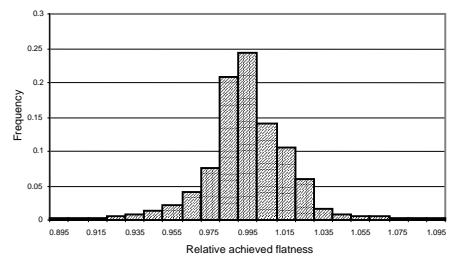


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PA Relative Achieved Flatness

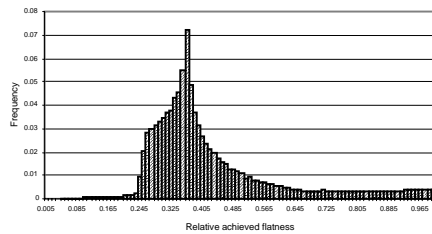


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RS Relative Achieved Flatness

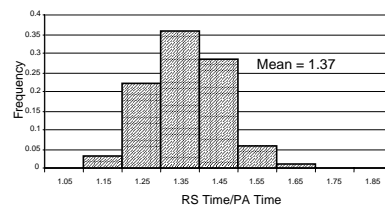


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Relative execution time (RS/PA)



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Conclusion

- PA produces (on average) $2/3^{\dagger}$ as many linear segments as RS[†]
- C-coded PA runs $37\%^{\dagger}$ faster than C-coded RS

[†] with flatness criterion being maintained within 4%
[†] results are insensitive to chosen flatness criterion



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